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# Numerical studies of internal solitary wave trains generated at edges in the topography.

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# 1 Executive summary

In this report some results from numerical studies of internal wave trains are given. The parameters of the experiments are chosen to be relevant for the Ormen Lange area. Many of the characteristics of the measured wave events are reproduced. However, the wave periods are still too short. In the model results the periods between consecutive waves in a wave train are typically 20 min whereas they are 45 min in the measurements we have focused on. A theoretical explanation for this discrepancy is given. Basically the periods between consecutive waves in a wave train depend on the distance from the generation point, and it would require more computer resources to follow the waves far enough to reproduce the 45 min periods in numerical experiments. The amplitudes of the waves are reduced with distance frome the shelf edge. This means that closer to the generation point, the wave amplitudes and corresponding velocities may be significantly larger than those measured.

## 2 Description of the numerical experiments

The $\sigma$-coordinate ocean model applied in the present studies is a two dimensional, $(x,z)$, version of the model described in [Berntsen(2000)] where $x$ and $z$ are the horizontal and vertical Cartesian coordinates respectively. The model is available from www.math.uib.no/BOM/. The variables are discretized on a C-grid. In the vertical, the standard $\sigma$-transformation, $\sigma = \frac{z-\eta}{H+\eta}$, where $\eta$ is the surface elevation, and $H$ the bottom depth, is applied. For advection of momentum and density a Total Variance Diminishing (TVD)-scheme with a superbee limiter described in [Yang and Przekwas(1992)] is applied in the present studies. The standard second order Princeton Ocean Model (POM) method is applied to estimate the internal pressure gradients ([Blumberg and Mellor(1987), Mellor(1996)]). The model is mode split with a method similar to the splitting described in [Berntsen et al.(1981)Berntsen, Kowalik, Sælid, and Sørli] and [Kowalik and Murty(1993)]. Even if the model is two dimensional, flow in the across domain direction is allowed. However, there will be no across domain variability in this flow. The effects of the earths rotation are taken into account in the present studies and the Coriolis parameter is set to $f = 1.2 \times 10^{-4}\,\mathrm{s}^{-1}$.

It is known that internal waves may be formed behind topographic features in a stratified ocean, see for instance [Gill(1982), Baines(1995), Kundu and Cohen(2004), Thorpe(2005)]. This is also studied recently in [Roth(2000)] and [Lamb(2007)]. In these papers it is shown that dispersive wave trains may be generated at a shelf edge in tidally forced systems. Inspired by the findings in [Roth(2000)] and [Lamb(2007)], a two-dimensional cross shelf model system has been set up with parameters of topography, forcing, and stratification that are relevant for the Ormen Lange area. The model area is 18000 m long. At $x = 0$ m a flow is forced into the system with a periodic forcing $u = U_0 \sin(\omega_f t)$ where $u$ is the velocity component in $x$ direction, $U_0$ the amplitude of the forced velocity. Generally the tidal signal is weak near the shelf edge in the Ormen Lange area. The wave trains were observed during periods with relatively strong winds. The wave trains are therefore assumed to be driven by inertial oscillations. The inertial frequency $\omega_f$ is approximately $0.00013\mathrm{s}^{-1}$ at our latitude, giving a period of 13.42 hours.

In one set of experiments the focus is on solitary wave propagation outside the shelf, but generated at the shelf edge. In these experiments the depth profile $H(x)$ in m, see Figure 1, is specified according to

$$H(x) = \begin{cases} -280 & , \quad x < 1000 \\ -400 + \frac{120}{1+((x-1000)/W_{slope})**4} & , \quad x > 1000 \,. \end{cases}$$

The slope width $W_{slope}$ is taken to be 500 m in the present experiments. The studies of wave propagation off the shelf edge are performed with $U_0$ equal to $0.3\,\mathrm{m\,s^{-1}}$ and $0.6\,\mathrm{m\,s^{-1}}$.

In another set of experiments the focus is on solitary wave propagation on the shelf. The depth profile in these studies $H(x)$ in m, see Figure 11, is specified according to

$$H(x) = \begin{cases} -280 & , \quad x > 1000 \\ -400 + \frac{120}{1+((x-1000)/W_{slope})**4} & , \quad x < 1000 \,, \end{cases}$$

where again the sill width is taken to be 500 m. The studies of wave propagation on the shelf are performed with $U_0$ equal to $0.3\,\mathrm{m\,s^{-1}}$ and $0.4\,\mathrm{m\,s^{-1}}$.

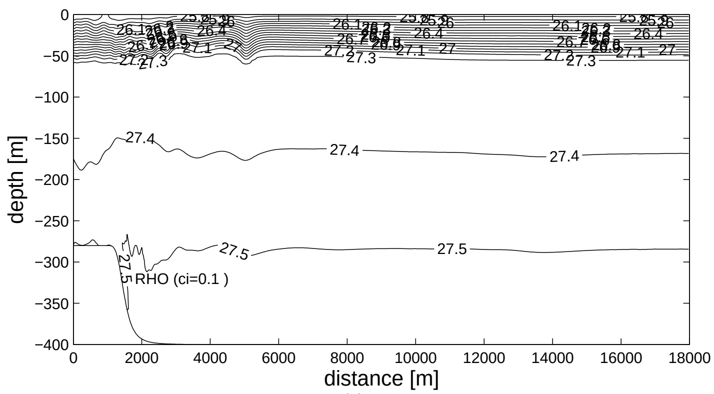
Based on the measurements provided from this area an idealised initial stratification is assumed. The density at the surface is $1025.7\,\mathrm{kg\,m^3}$, the density at 50 m depth is $1027.3\,\mathrm{kg\,m^3}$, and at 400 m depth the density is taken to be $1027.6\,\mathrm{kg\,m^3}$. Linear interpolation is used to compute the full density profile. With this density profile and theory about internal wave propagation the wave speed may be estimated to be in the range from 0.63 to $0.88\,\mathrm{m\,s^{-1}}$.

The model grid resolution is 25 m horizontally and 100 equidistant layers are used vertically. The model is run for two inertial periods. The results from the four experiments are summarised in the following sections.

## 3 Solitary wave propagation off the shelf ($U_0$ = 0.3 m s$^{-1}$)

Below are some results for the case with solitary waves off the shelf, using a maximum inflow velocity of 0.3 m s$^{-1}$. A section of the density field after 11.74 hours and the depth profile is given in Figure 1. To get a better impression of the wave propagation, the density fields and two components of the velocity field at three consecutive times are given in Figures 2, 3, and 4. In these figures the focus is on the internal waves. At time equal to 11.74 hours only one wave of suppression is seen in Figure 2. This wave is also seen after 13.42 hours and 15.09 hours. However, at these later times a group of solitary waves is seen trailing the leading wave, In Figure 5 the $u$ components at $x$ = 5000 m and $x$ = 15000 m are plotted as functions of time and depth, again with focus on the passing train of solitary waves.

The speed of the front of the wave train in the period from 13.42 hours to 15.09 hours is approximately 0.56 m s$^{-1}$. The wave train changes form, and develops the typical solitary wave train characteristics as it propagates away from the edge. From Figure 5 the period between consecutive waves in the group may be estimated to be 24 minutes. It is also clearly seen that the first wave in the train is strongest, and the amplitudes of the waves behind are gradually reduced.



(a)

Figure 1: Topography and density field after 11.74 hours (7/8 of an inertial periods).

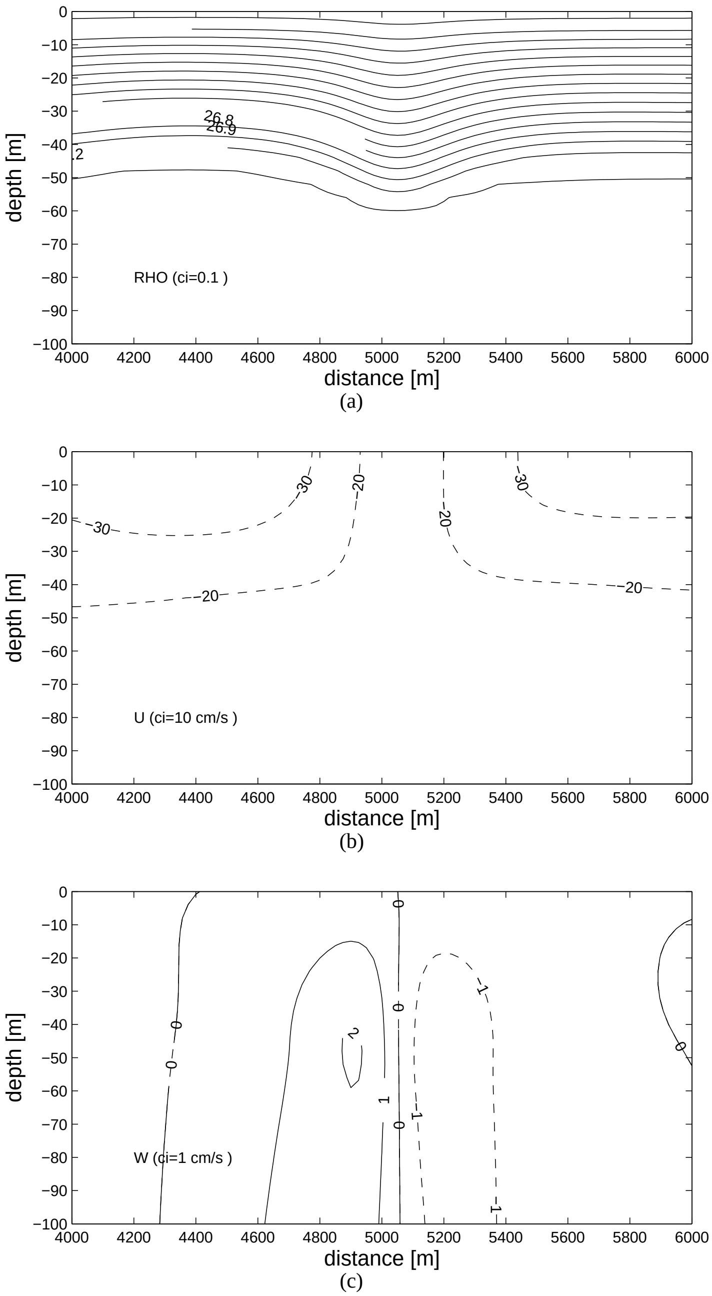
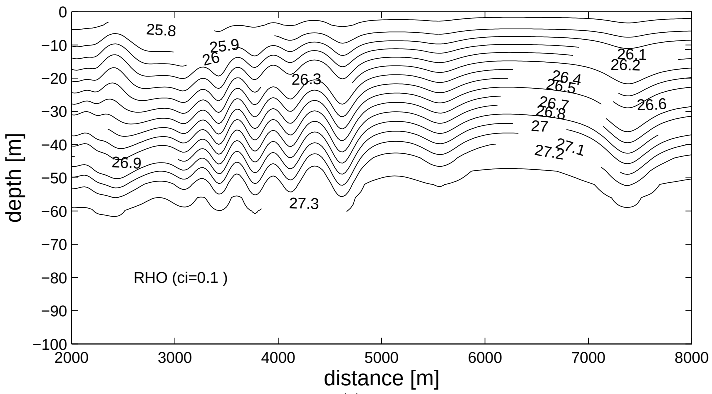



Figure 2: The train of solitary waves after 11.74 hours. The density field is given in a), the *u* component of the velocity field is given in b), and the vertical velocities in c).

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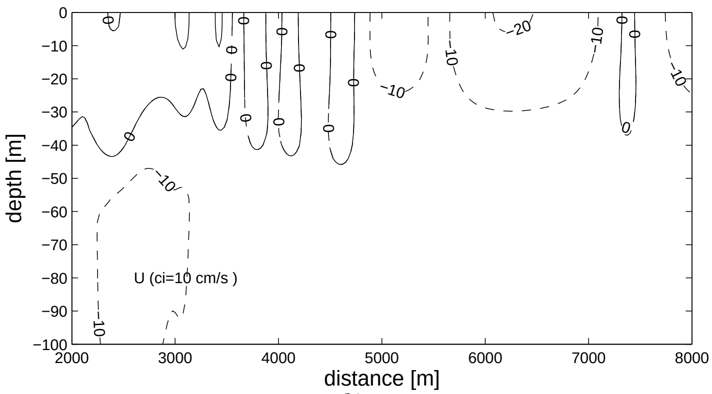


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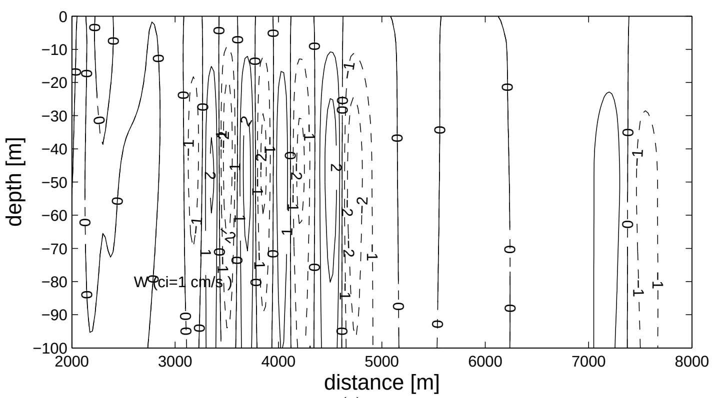



(c)

Figure 3: The train of solitary waves after 13.42 hours. The density field is given in a), the *u* component of the velocity field is given in b), and the vertical velocities in c).

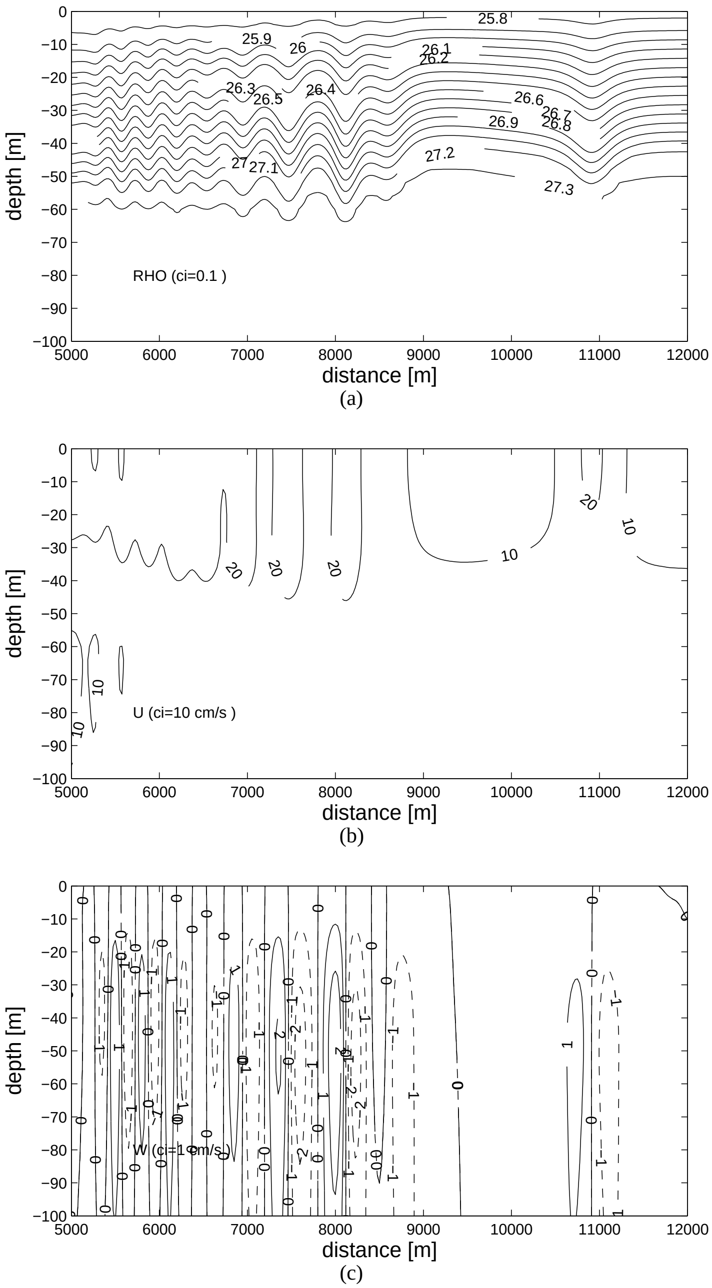
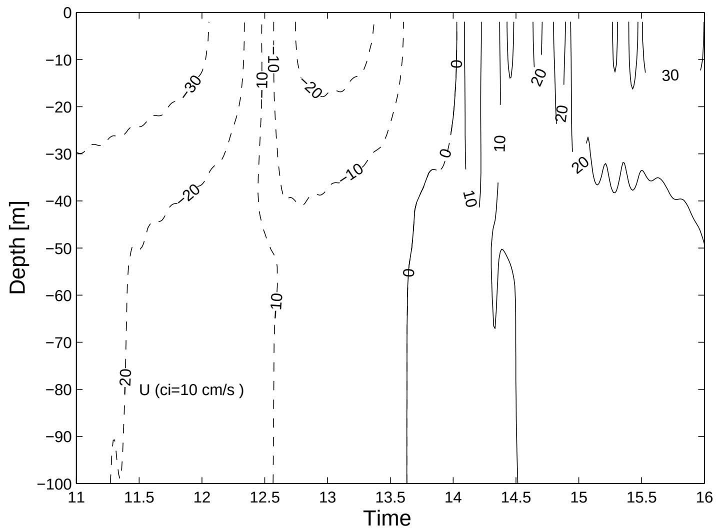



Figure 4: The train of solitary waves after 15.09 hours. The density field is given in a), the *u* component of the velocity field is given in b), and the vertical velocities in c).

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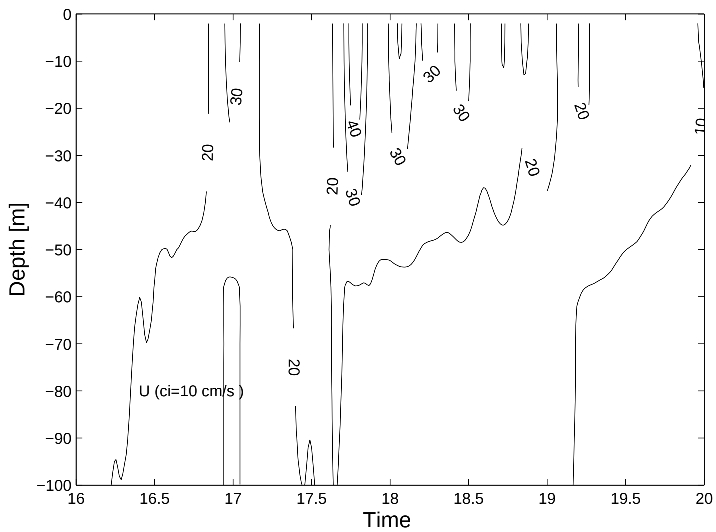


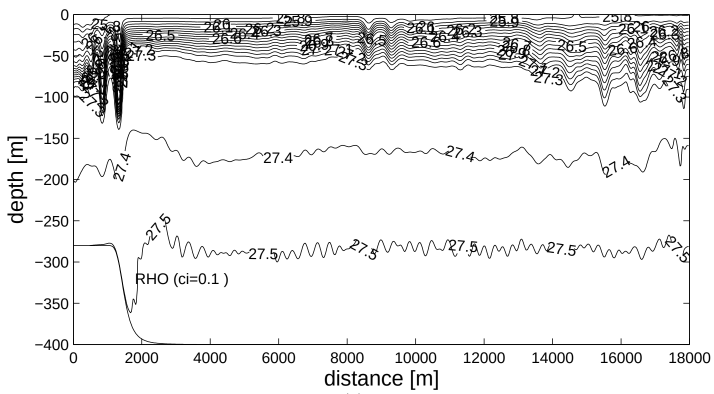

(b)

Figure 5: Time development of the vertical profile of along section velocities at $x = 5000$ m (top) and $x = 15000$ m with focus on the waves. (The time is in hours.)

# 4 Solitary wave propagation off the shelf ($U_0 = 0.6\,\mathrm{m\,s^{-1}}$)

Below are some results for the case with solitary waves off the shelf, using a maximum inflow velocity of $0.6\,\mathrm{m\,s^{-1}}$. A section of the density field after 13.42 hours and the depth profile is given in Figure 6. The solitary wave trains generally become much stronger when the inflow velocity is increased. The density fields and two components of the velocity field at three consecutive times are given in Figures 7, 8, and 9. In these figures the focus is on the solitary wave train. In Figure 10 the $u$ components at $x = 5000$ m and $x = 15000$ m are plotted.

The speed of the front of the wave train in the period from 13.42 hour to 15.09 hours is approximately $0.57\,\mathrm{m\,s^{-1}}$ and in the period from 15.09 hours to 16.78 hours approximately $1.09\,\mathrm{m\,s^{-1}}$. The wave group also changes form, and develops the typical solitary wave train characteristics as it propagates away from the edge. From Figure 10 the period between consecutive waves in the group may be estimated to be 15 minutes. It is also clearly seen that the first wave in the group is strongest, and the amplitudes of the waves behind are gradually reduced. The speed in the firts wave exceeds $1.00\,\mathrm{m\,s^{-1}}$.




(a)

Figure 6: Topography and density field after 13.42 hours (one inertial period).

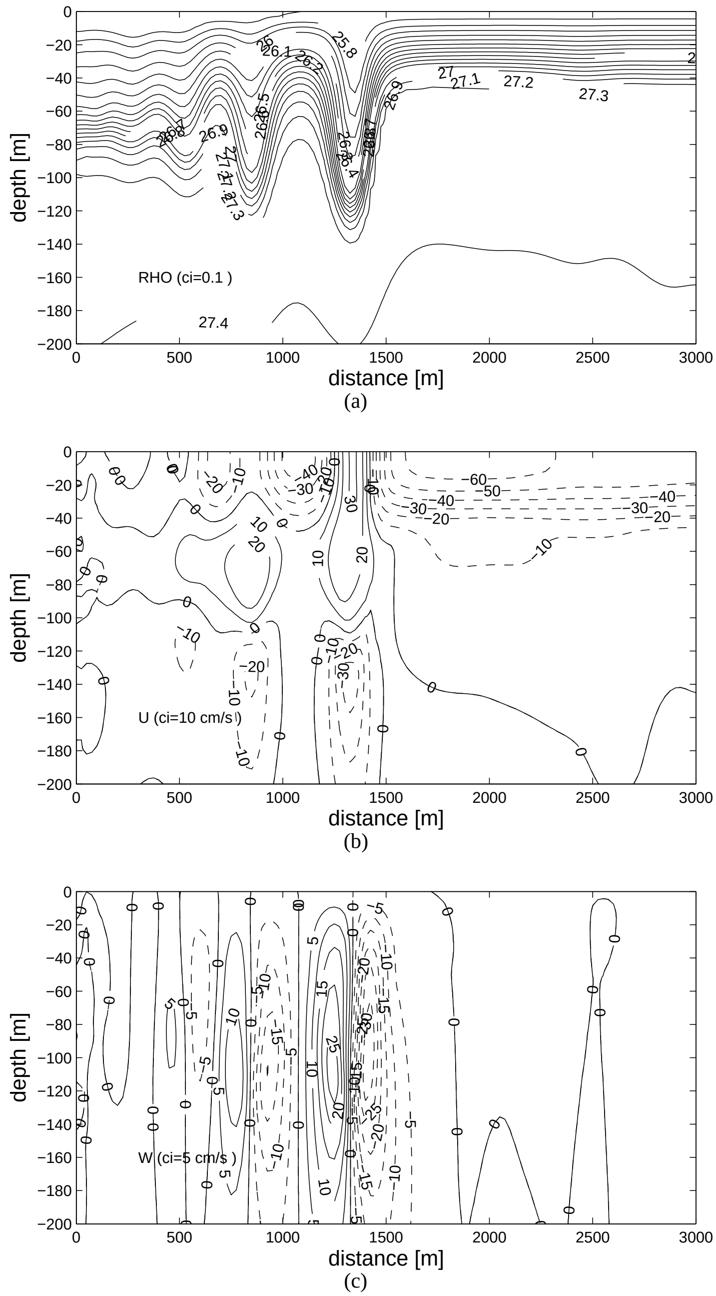



Figure 7: The train of solitary waves after 13.42 hours. The density field is given in a), the *u* component of the velocity field is given in b), and the vertical velocities in c).

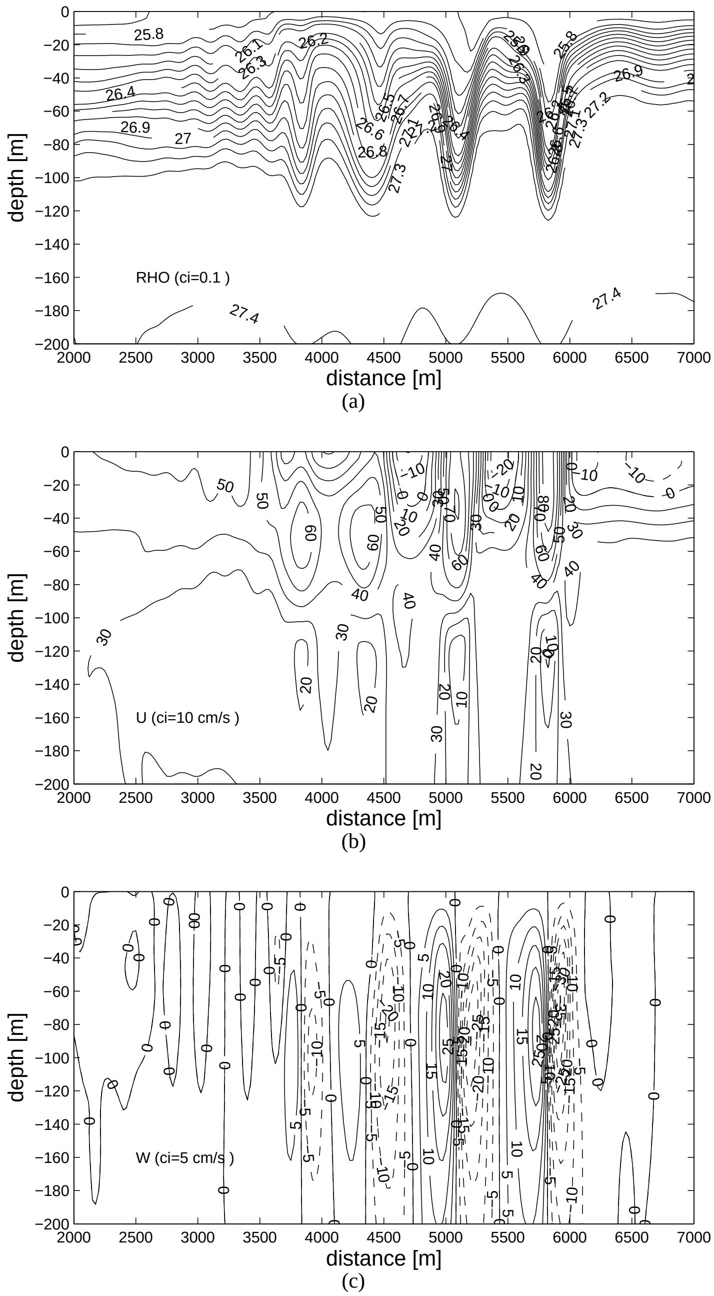
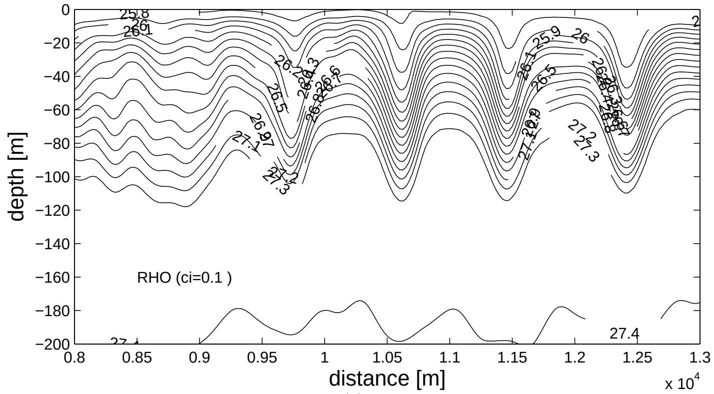



Figure 8: The train of solitary waves after 15.09 hours. The density field is given in a), the *u* component of the velocity field is given in b), and the vertical velocities in c).



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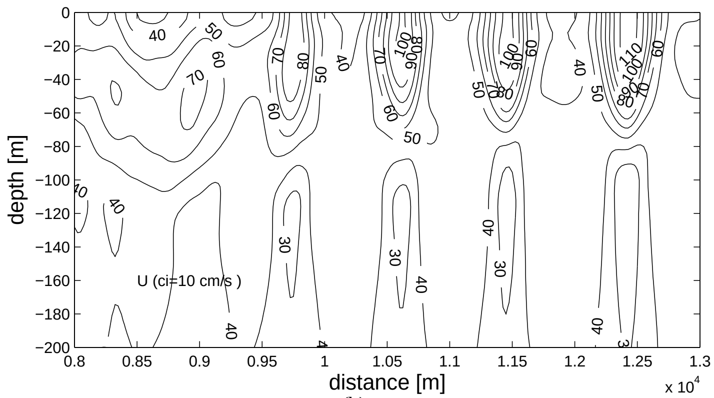

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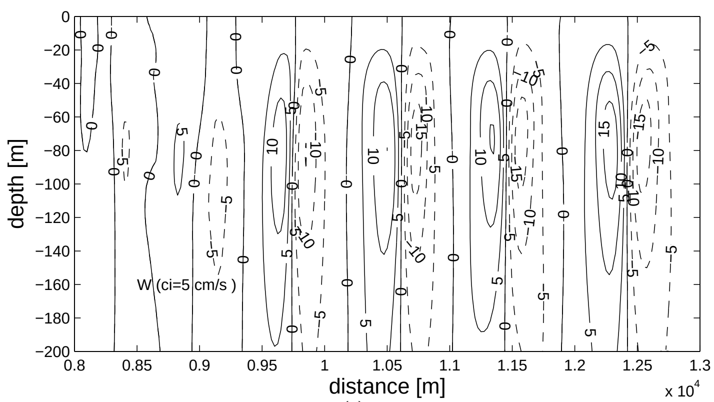
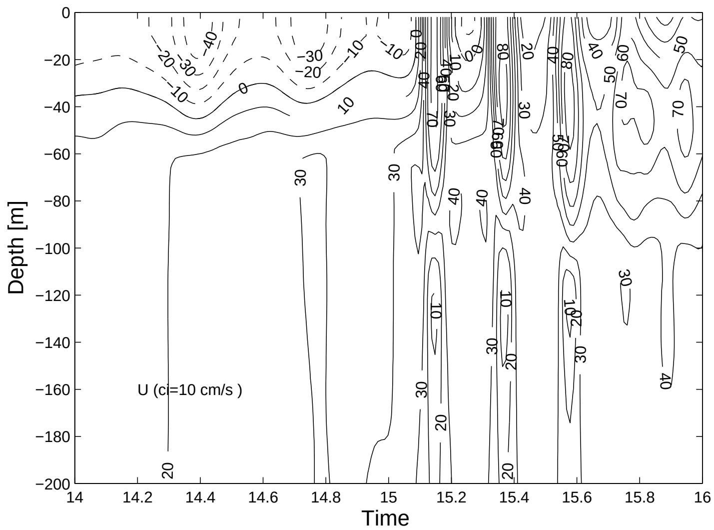


(c)

Figure 9: The train of solitary waves after 16.78 hours. The density field is given in a), the *u* component of the velocity field is given in b), and the vertical velocities in c).

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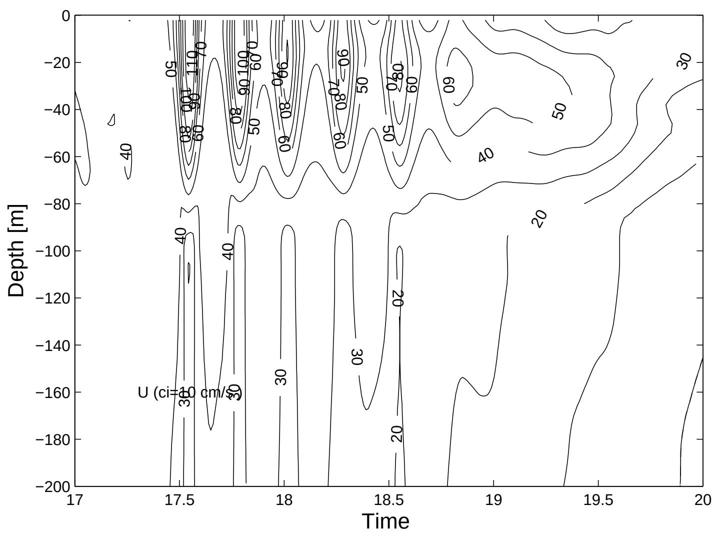


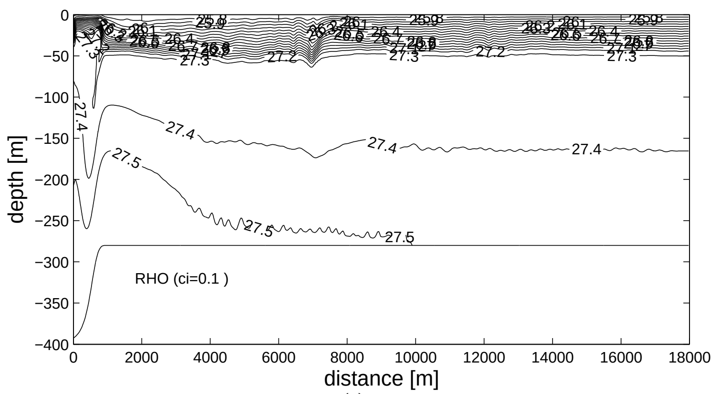

(b)

Figure 10: Time development of the vertical profile of along section velocities at $x = 5000$ m (top) and $x = 15000$ m with focus on the waves.

## 5 Solitary wave propagation on the shelf ($U_0$ = 0.3 m s$^{-1}$)

Below are some results for the case with solitary waves on the shelf, using a maximum inflow velocity of 0.3 m s$^{-1}$. A section of the density field after 11.74 hours and the depth profile is given in Figure 11. The density fields and two components of the velocity field at three consecutive times are given in Figures 12, 13, and 14. In Figure 15 the $u$ components at $x$ = 10000 m and $x$ = 15000 m are plotted.

The speed of the front of the wave train in the period from 11.74 hour to 13.42 hours is approximately 0.53 m s$^{-1}$ and in the period from 13.42 hours to 15.09 hours approximately 0.68 m s$^{-1}$. The wave group also changes form, and develops the typical solitary wave train characteristics as it propagates away from the edge. From Figure 15 the period between consecutive waves in the group may be estimated to be 24 minutes.




(a)

Figure 11: Topography and density field after 11.74 hours (7/8 of an inertial periods).

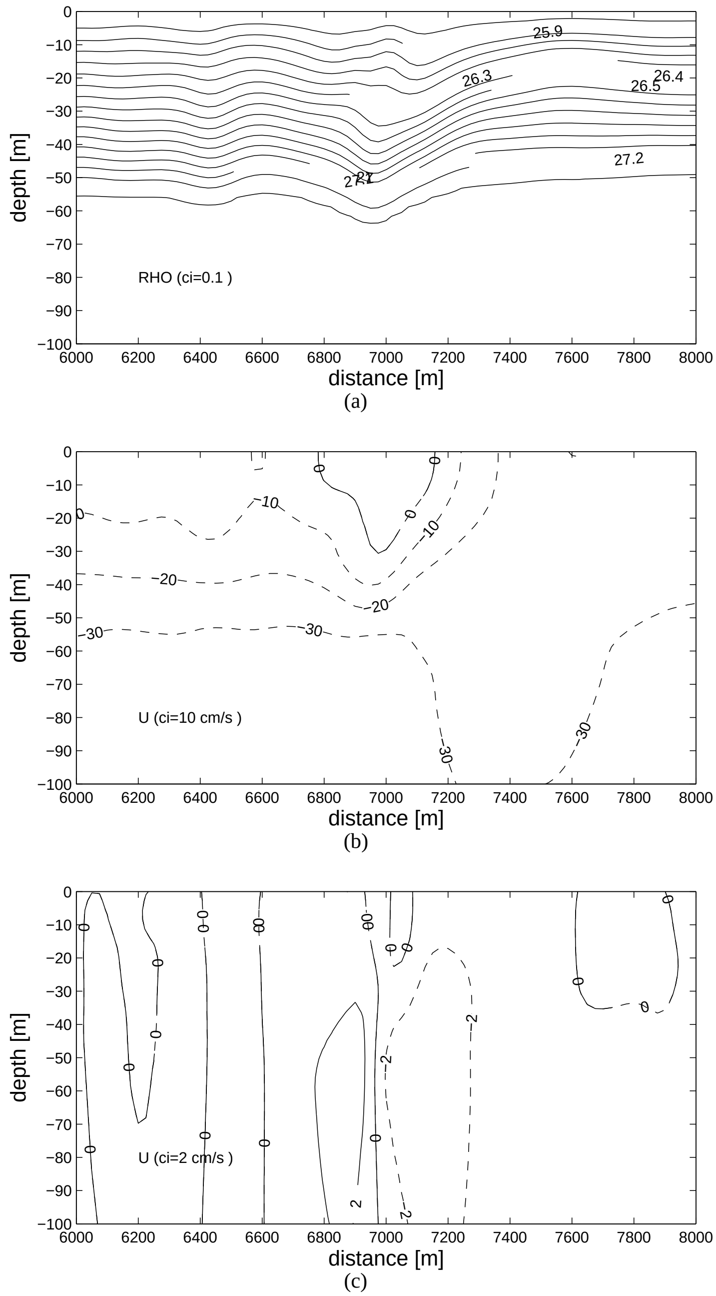



Figure 12: The train of solitary waves after 11.74 hours. The density field is given in a), the *u* component of the velocity field is given in b), and the vertical velocities in c).

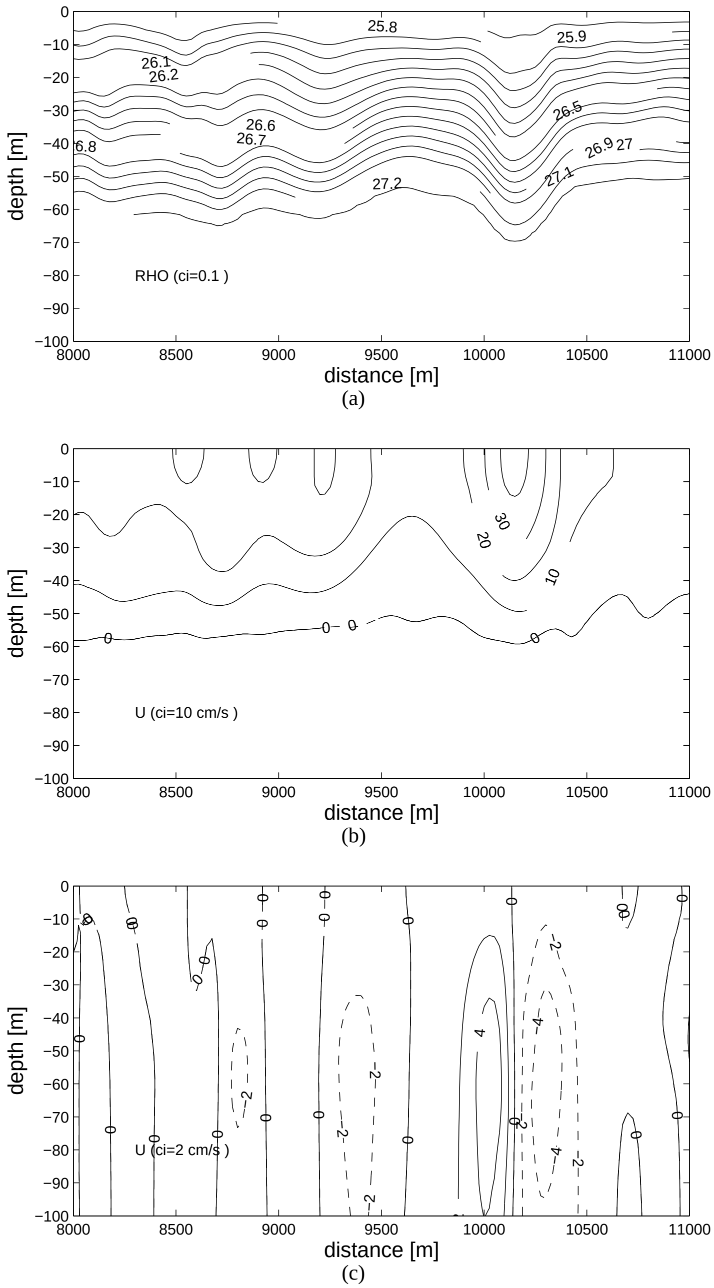



Figure 13: The train of solitary waves after 13.42 hours. The density field is given in a), the *u* component of the velocity field is given in b), and the vertical velocities in c).

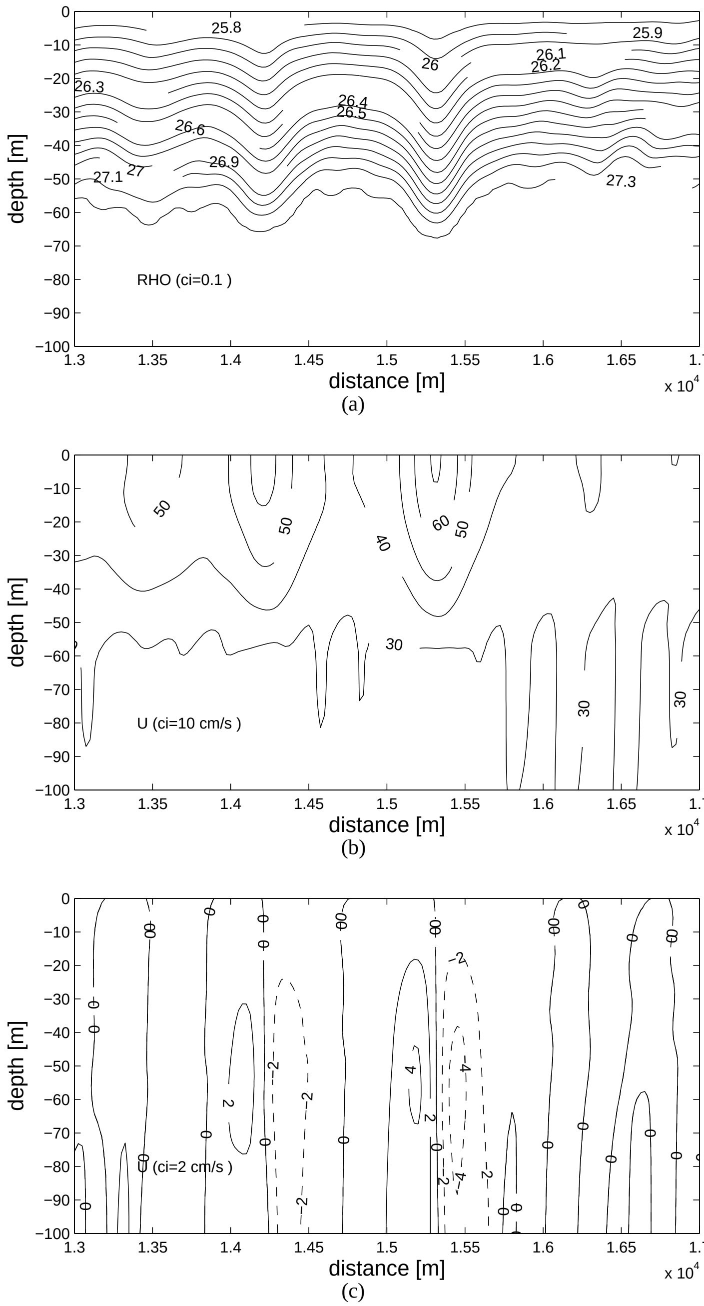



Figure 14: The train of solitary waves after 15.09 hours. The density field is given in a), the *u* component of the velocity field is given in b), and the vertical velocities in c).

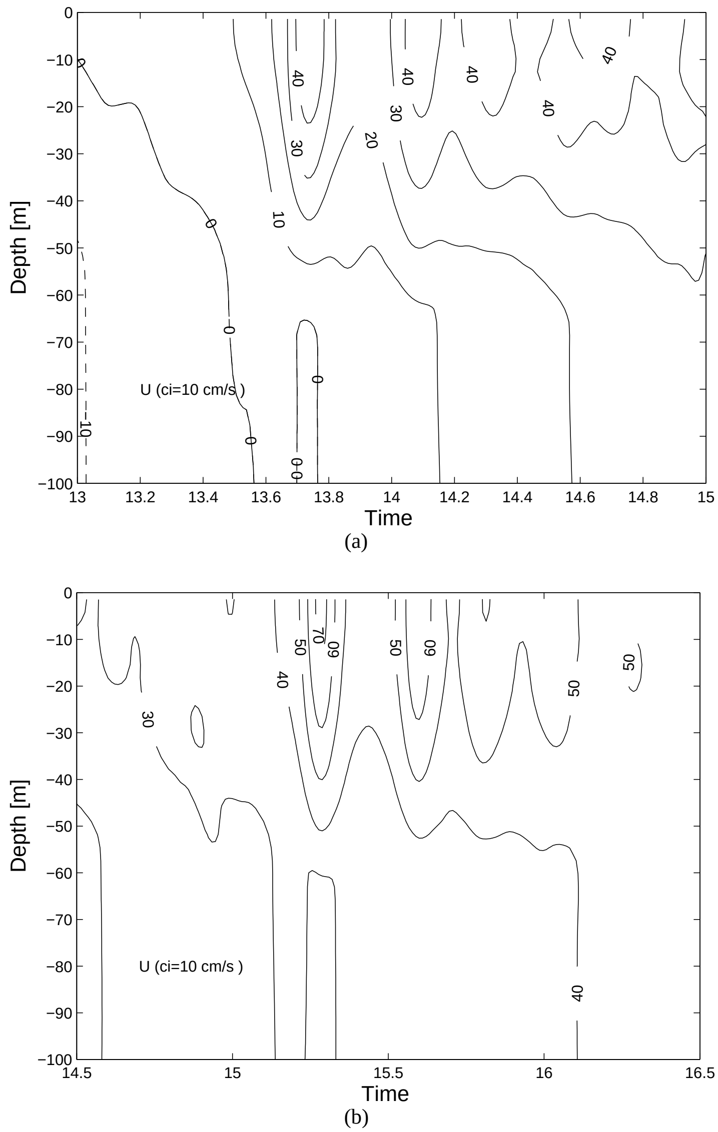
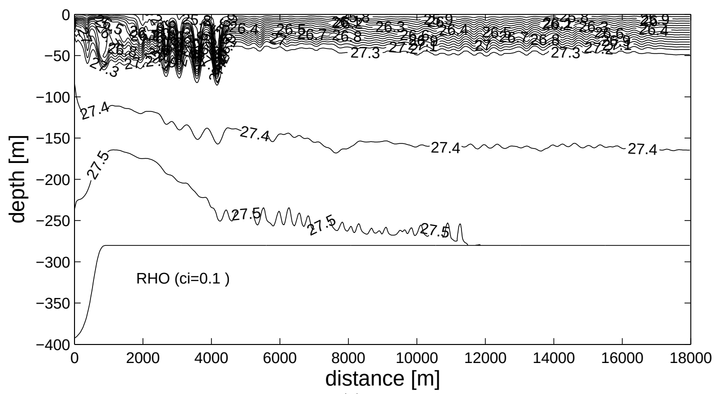



Figure 15: Time development of the vertical profile of along section velocities at $x$ = 10000 m (top) and $x$ = 15000 m with focus on the waves.

# 6 Solitary wave propagation on the shelf with stronger inflow ($U_0$ = 0.4 m s$^{-1}$)

Below are some results for the case with solitary waves on the shelf, using a stronger forcing. Generally the solitary waves develops earlier and are seen much more clearly, see below a section of the density field after 11.74 hours. The density fields and two components of the velocity field at three consecutive times are given in Figures 17, 18, and 19. In these figures the focus is on the solitary wave train. In Figure 20 the $u$ components at $x$ = 5000 m and $x$ = 15000 m are plotted, again with focus on the passing group of solitary waves.

The speed of the front of the wave train in the period from 11.74 hour to 13.42 hours is approximately 0.63 m s$^{-1}$ and in the period from 13.42 hours to 15.09 hours approximately 1.0 m s$^{-1}$. Thus as in the more weakly forced case in the previous section, the wave group gains speed away from the edge. From Figure 20 the period between consecutive waves in the group may be estimated to be 20 minutes. It is again clearly seen that the first wave in the group is strongest, and the amplitudes of the waves behind are gradually reduced.




(a)

Figure 16: Topography and density field after 11.74 hours (7/8 of an inertial periods).

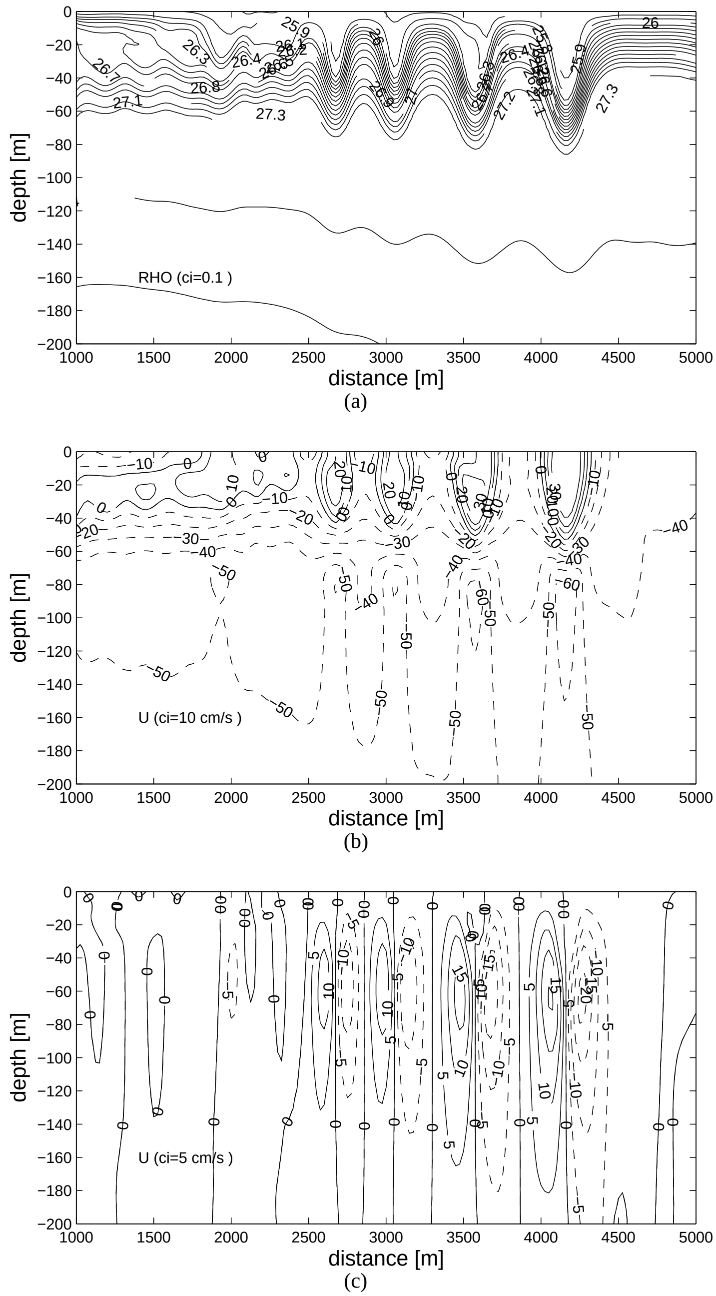



Figure 17: The train of solitary waves after 11.74 hours. The density field is given in a), the *u* component of the velocity field is given in b), and the vertical velocities in c).

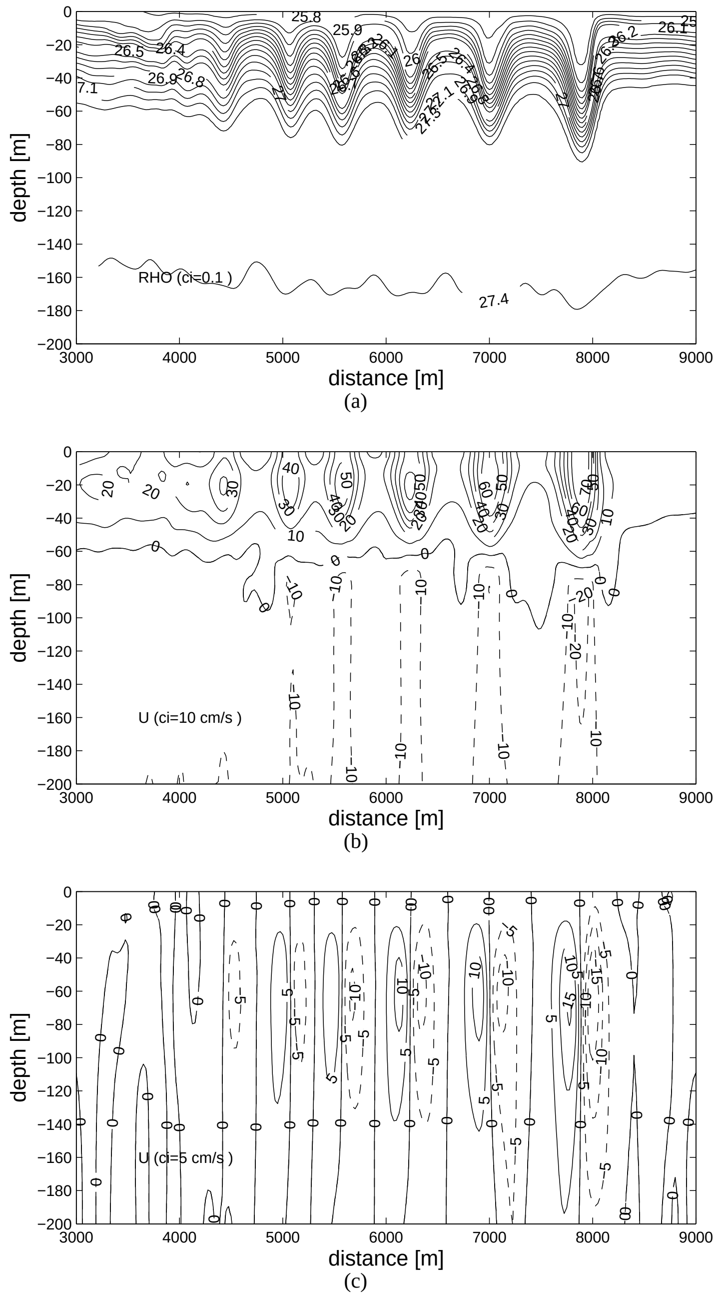



Figure 18: The train of solitary waves after 13.42 hours. The density field is given in a), the *u* component of the velocity field is given in b), and the vertical velocities in c).

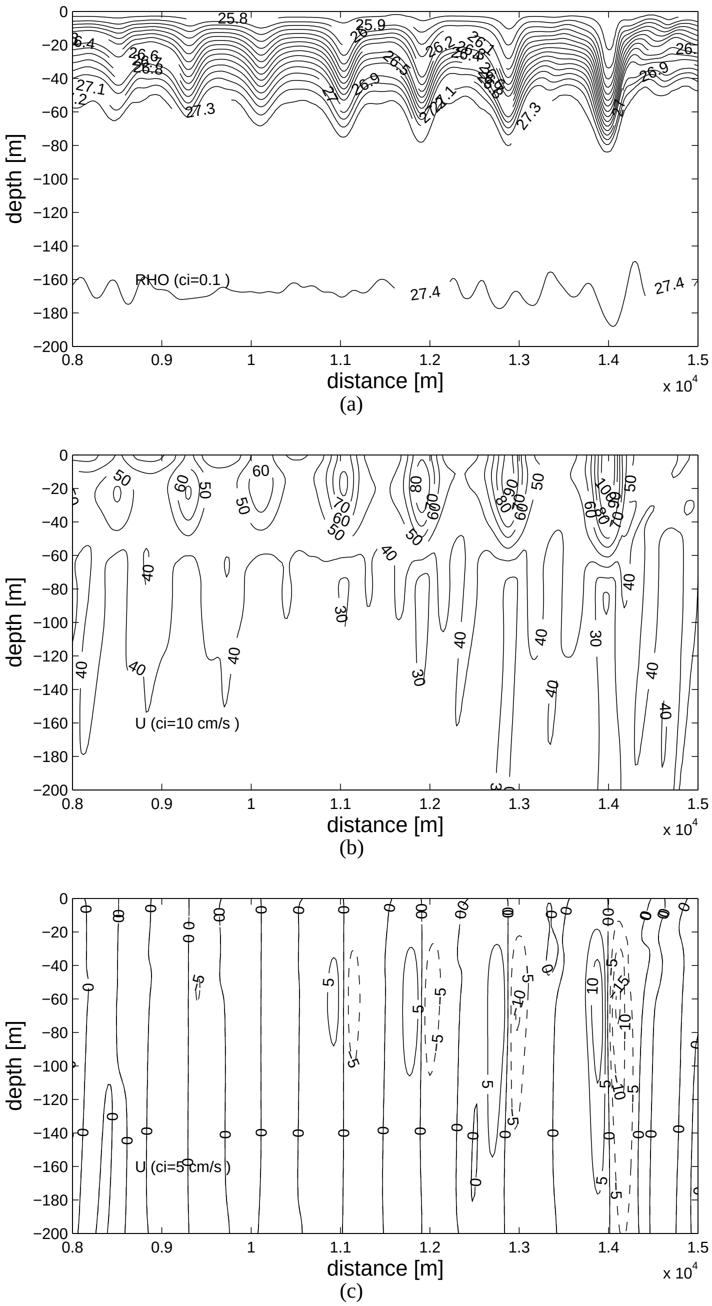
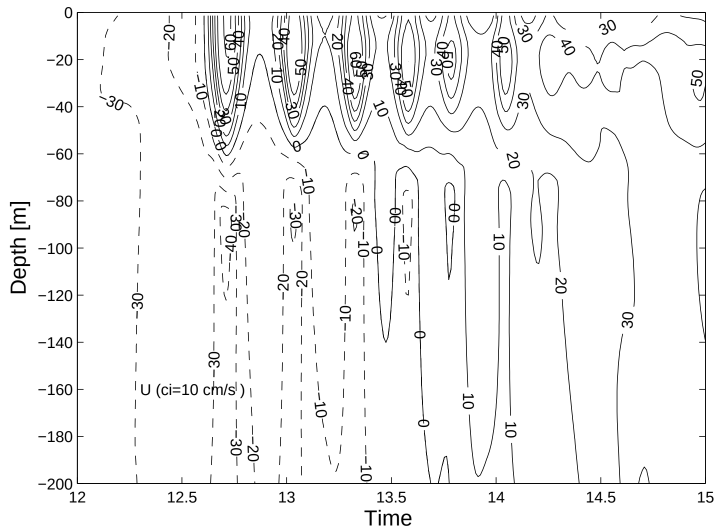
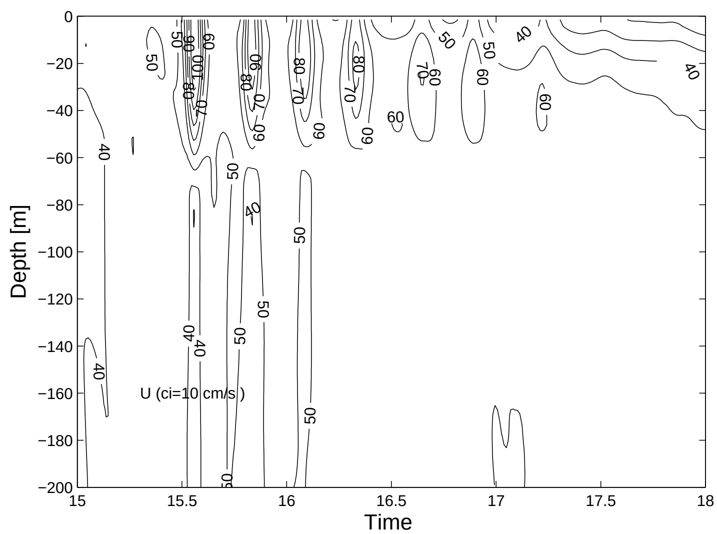



Figure 19: The train of solitary waves after 15.09 hours. The density field is given in a), the *u* component of the velocity field is given in b), and the vertical velocities in c).




(a)




(b)

Figure 20: Time development of the vertical profile of along section velocities at $x = 5000$ m (top) and $x = 15000$ m with focus on the waves.

# 7 Discussion

Wave phenomena with periods of approximately 45 min are observed near the shelf edge in the Ormen Lange area. They are characterised by one leading wave with 4 to 5 gradually weaker waves behind it. The observed waves thus have the characteristics of internal solitary wave trains in stratified oceans. The tides are generally weak in this area. Therefore, wind and inertial oscillations are taken as forcing in the numerical experiments reported here. See the description of the general oceanography in this area [Mathisen et al.(2007)Mathisen, Berntsen, and Furnes].

It is known that accelerating flow over an edge easily starts off a train of solitary waves in a stratified ocean. These waves may have wave lengths of approximately 500 to 1000 m and grid sizes of approximately 25 m are necessary to resolve them in numerical studies. This means that they are not captured in general studies of the circulation in this area, and the strong current events, with speeds that my exceed $1\,\mathrm{m\,s^{-1}}$, associated with the waves are not captured. In the preliminary studies reported here current speeds that exceeds $1.10\,\mathrm{m\,s^{-1}}$ are found without increasing the background inflow velocities towards the maximum values measured along the shelf edge. With stronger forcing, the wave trains develop earlier, and they get larger amplitudes. However, the periods between consecutive waves in a wave train seem to be reduced as the forcing is increased. With stronger forcing, the speed of the wave train also seems to increase with distance from the generation point.

The waves gradually changes shape as they propagate. In the present studies they are still fairly strong 17 km away from the generation point, which is as far as they have been 'followed' using simulations that take approximately 15 hours each.

Basically the waves produced in these numerical studies have many of the characteristics of the observed waves. The major discrepancy is the period, which is approximately 20 min in the model results and 45 min in the observations. It may be noted that periods of 20 min are also measured over the continental shelf off Oregon, see [Moum and Smyth(2006)].

The periods between consecutive waves in a wave train generated by a strong pulse at for instance an edge in the topography may, however, change with travel time or with distance from the generation point, see [Dysthe(2004), Whitham(1974)].

For surface wave trains generated by a strong initial water elevation, the surface elevations $\eta(x,t)$ may be given by

$$\eta(x,t) \sim \frac{1}{2Ct^{1/3}} Ai\left(\frac{x-\sqrt{gh}}{Ct^{1/3}}\right),$$

where $x$ is the horizontal coordinate, $t$ is time, $g$ is the gravity, $h$ is the water depth, $Ai$ is the Airy function, and $C = (\frac{h^2\sqrt{gh}}{2})^{1/3}$, see [Dysthe(2004)]. For an internal solitary wave train, the speed $\sqrt{gh}$ must be replaced with $\sqrt{g'h'}$ where $g' = g\frac{\Delta\rho}{\rho_0}$ and $\Delta\rho$ is the density difference and $\rho_0$ the background density. Furthermore, $h$ must be replaced with $\frac{h_1 h_2}{h_1+h_2}$ where $h_1$ is the depth of the surface layer and $h_2$ the thickness of the bottom layer in a two layer system.

The solution given above is for a more simple system than what we have in the Ormen Lange area, and for a more idealised initial state than what we have in the numerical simulations. However, the solution has many of the characteristics of both the observed waves and the modelled waves, with one strong leading wave and decaying trailing waves behind it. The factor $t^{1/3}$ in the denominator of the argument to the Airy function also indicate that the period between consecutive waves in a wave train may be doubled, if $t$ is increased with a factor 8. Since the waves travel with almost constant speed, this indicate that if the period is 20 min 10 km from the generation point, or the shelf edge, the period will be 40 min 80 km from the generation point. Also the amplitudes of the waves will be reduced as they move. In essence this also means that if the observed periods of the waves are the same for all events, it is lightly that the waves come from the same generation point, or from generation points equally far from the observation point. The factor $t^{1/3}$ in the denominator of the amplitude of the Airy function means that closer to the shelf edge, the current velocities may be substantially larger.

We would also like to point at the papers on Waves in Geophysical Fluids in [Grue and Trulsen(2006)] including [Grue(2006)], [Kharif and Pelinovsky(2006)], and [Morozov(2006)]. In [Kharif and Pelinovsky(2006)] dispersion enhancement of transient wave packets is discussed and an Airy function solution of the type given above is presented. In [Morozov(2006)] also internal waves in the Arctic region are described.

A full explorations of these wave trains, including sensitivity to the parameters involved (forcing, period forcing, topography, stratification), investigations of possible maximum speeds, and studies where we follow the waves further is a full project in itself. The waves are followed 18 km in the present studies, and approximately 100 km is possible using more computer resources. It could also be mentioned that even if the focus in this report has been on solitary wave trains outside mid-Norway such waves are observed many places in the ocean, including elsewhere in Norwegian waters, see [Global Ocean Associates(2004)].

**Acknowledgement.** Many thanks to Kristian B. Dysthe for pointing us the theory on wave fronts.

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